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# Sound Generation by Impulse Excited Plates Coupled to Acoustic Cavities

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**Abstract**
This paper is concerned with vibroacoustics in the time domain. One of the aims is to compare results given by an semi-analytical technique based on the resonance modes with a finite difference technique. An other goal is to describe the response of a fluid-loaded plate (displacement of the structure and sound pressure in the fluid) coupled to a rigid cavity when it is excited by a Ricker wavelet and to see the influence of the excitation on the response of system.

## INTRODUCTION

The amount of publications on fluid/structure interactions over the past three decades is quite impressive. The earliest one is due to M.C. JUNGER and D. FEIT [1], the first edition of which appeared in 1972. Then a couple of books have been published in the eighties [2, 3]. These books provide a very complete overview of the knowledge in the domain. Let us also mention the CISM Course [4] in which the basic concepts of the fluid/structure interaction phenomena are described. Most of the studies are conducted for harmonic time dependence because of its relative simplicity that allows to give analytical or asymptotic expansion. When explicit time dependence is needed, less analytical work had been done, most of it in underwater acoustics see for example [5] ; roughly, most of the methods are based on a numeric inversion of the harmonic equations by Fourier integrals. An other way to solve time domain problems is to use a direct approximation of the equation based on finite difference techniques. This paper shows how finite differences methods and analytical methods can be used to compute sound generation by impulse excited plates coupled to acoustic cavities. And, in particular how both methods are able to describe the radiation of subsonic and supersonic plate waves.

Section 2 is devoted to the description of the equations.

In section 3, it is shows how the resonance modes can be used to describe the response of a fluid-loaded structure (displacement of the structure and sound pressure in the fluid). First, the response of the system to a harmonic excitation is expanded into a series of eigenmodes which depend on frequency. By using a Fourier inverse transform, it is then possible to express the response of the system to a transient excitation in terms of the fluid-loaded structure resonance modes. These modes do not depend on frequency. When the resonance modes are known, the response of the system to any kind of excitation is obtained. It is show how a perturbation expansion that account for a "light" fluid loading allows to compute these resonance modes in a very simple way.

In section 4, the equations are solved by a finite difference technique.

## GENERAL STATEMENT OF THE PROBLEM

In the rest of this paper, small case letters correspond to time functions while capital letters denote time harmonic functions with time dependence $\exp(-\imath\omega t)$. Let us consider a rectangular domain $\Sigma$ defined in cartesion co-ordinate $(O, x, z)$ by $x \in [0, L_x] \times z \in [0, L_z]$. $\partial\Sigma$ is the boundary of $\Sigma$. A thin monodimensionnal plate of thickness $h$, length $L_x$ and simply supported at its boundary, occupies the upper surface of this domain. This plate closes a rigid rectangular cavity lies in $\Sigma$. The mechanical and geometric parameters of the plate are Young's modulus $E = 200GPa$, Poisson's coefficient $\nu = 0.3$, density $\rho_p = 7800Kg/m^3$, $h = 5mm$ and $L_x = 1m$. The plate is damped by a proportional damping given by $\eta_p = 1$. The geometry of the cavity is given by $L_x = 1m$ and $L_z = 0.8m$. The fluid inside the cavity is air, considered as a perfect gas, of density $\rho_p = 1,2Kg/m^3$ and sound speed celerity $c_f = 340m/s$. The critical frequency of this plate is 2517 Hz.

The plate is excited by a point-mecanical force $f(x,t)$ located at $x_e$ of unit maximum amplitude. Its time dependence is that of a Ricker wavelet. It is defined as the second time derivative of the gaussian function $g(t) = \exp(-\pi^2(1-f_0t)^2)$. That is $f(x,t) = \delta_{x_e}(x)g''(t)/g''(1/f_0) = \delta_{x_e}(x)(1-2\pi^2(1-f_0t)^2)\exp(-\pi^2(1-f_0t)^2)$. One can consider that this signal is of finite duration between 0 and $Te = 2/f_0$ and of finite spectrum between 0 and $3f_0$.

Let us denote by $u(x,t)$, the normal displacement of the plate $v(x,t)$ its velocity. $u(x,t)$ is described by the usual Kirchhoff equation. In the cavity, the acoustic pressure, denoted by $p(x,z,t)$, is governed by the d'Alembert equation. On the boundary of the cavity, a Neumann condition is imposed. $u(x,t)$ and $p(x,z,t)$ satisfy the following system of equations:

$$D\Delta^2u(x,t) + \rho_p h\ddot{u}(x,t) + \eta_p\dot{u}(x,t) = f(x,t) - p(x,0,t) \tag{1}$$
$$\Delta p(x,z,t) - 1/c_f^2\ddot{p}(x,z,t) = 0 \tag{2}$$
$$u(0,t) = 0, u''(0,t) = 0, u(l_x,t) = 0, u''(L_x,t) = 0 \tag{3}$$
$$u(x,0) = 0, \dot{u}(x,0) = 0 \tag{4}$$
$$\partial_n p(x,z,t) = 0 \text{ on } \partial\Sigma \tag{5}$$
$$p(x,z,0) = 0, \dot{p}(x,z,0) = 0 \tag{6}$$
$$\partial_n p(x,z,t) = \rho_f\ddot{u}(x,t) \text{ on the plate,} \tag{7}$$

where $D = Eh^3/(12(1-\nu^2))$ is the bending rigidity if the plate. In these equations $\dot{p}$ or $\dot{u}$ stand for the time derivative and $\partial_n$ is the normal derivative. Equations (3,5) are the boundary conditions for the displacement and the acoustic sound pressure. Equations (4,6) are the initial cauchy conditions for the displacement and the acoustic sound pressure. The last equation (7) ensures the continuity of the accelerations of both the plate and pressure inside the cavity.

## MODAL EXPANSIONS AND TIME DOMAIN SOLUTION

In the harmonic regime, one has

$$D\Delta^2U(x,\omega) - \rho_p h\omega^2U(x,\omega) - \imath\omega\eta_pU(x,\omega) = F(x,\omega) - P(x,0,\omega) \tag{8}$$
$$\Delta P(x,z,\omega) + \omega^2/c_f^2P(x,z,\omega) = 0 \tag{9}$$
$$U(0,\omega) = 0, U''(0,\omega) = 0, U(l_x,\omega) = 0, U''(L_x,\omega) = 0 \tag{10}$$
$$\partial_n P(x,z,\omega) = 0 \text{ on } \partial\Sigma \tag{11}$$
$$\partial_n P(x,z,\omega) = -\rho_f\omega^2(x,\omega) \text{ on the plate.} \tag{12}$$

This system of equations for $U$ and $P$ can be brought back to an integrodifferential equation for $U$ only by using Green's representation of the pressure. $G(x,z,x',z',\omega)$, the Green's function of the rigid cavity is given by the modal expansion

$$G(x,z,x',z',\omega)=\frac{c_f^2}{L_xL_z}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\epsilon_m\epsilon_n\frac{\Psi_{mn}(x,z)\Psi_{mn}(x',z')}{\omega^2-\omega_{mn}^{02}},$$

where $\omega_{mn}^0=c_f\sqrt{\left(\frac{m\pi}{L_x}\right)^2+\left(\frac{n\pi}{L_z}\right)^2}$ is the mn-th eigenpulsation, $\epsilon_m$ is the Neumann factor, that is $\epsilon_m=1$ if $m=0$ and $\epsilon_m=1$ if $m\neq 0$ and $\Psi_{mn}(x,z)=\cos(m\pi x/L_x)\cos(n\pi z/L_z)$ is the mn-th eigenmode of the rigid cavity. The pressure inside the cavity is given by the integral equation

$$P(x,z,\omega)=\omega^2\rho_f\int_0^{L_x}U(x',\omega)G(x,z,x',0,\omega)\mathrm{d}x'. \tag{13}$$

Introducing this result in equation(8), one obtains

$$D\Delta^2U(x,\omega)-\rho_ph\omega^2\left(U(x,\omega)+\imath\frac{\epsilon_p}{\omega}U(x,\omega)-\epsilon_f\int_0^{L_x}U(x',\omega)G(x,0,x',0,\omega)\mathrm{d}x'\right)=F(x,\omega)$$
$$U(0,\omega)=0,U''(0,\omega)=0,U(l_x,\omega)=0,U''(L_x,\omega)=0,$$

where $\epsilon_p=\eta_p/\rho_ph$ and $\epsilon_f=\rho_f/\rho_ph$ are small parameters for a slightly damped plate in air. Now, one defines the eigenmodes $\tilde{U}_l(x,\omega)$ and the eigenpulsations $\tilde{\omega}_l(\omega)$ of this problem as the non zero solutions of

$$D\Delta^2\tilde{U}_l(x,\omega)-\rho_ph\tilde{\omega}_l^2(\omega)\left(\tilde{U}_l(x,\omega)+\imath\frac{\epsilon_p}{\omega}\tilde{U}_l(x,\omega)-\epsilon_f\int_0^{L_x}\tilde{U}_l(x',\omega)G(x,0,x',0,\omega)\mathrm{d}x'\right)=0,$$
$$\tilde{U}_l(0,\omega)=0,\tilde{U}_l''(0,\omega)=0,\tilde{U}_l(l_x,\omega)=0,\tilde{U}_l''(L_x,\omega)=0.$$

It is easy to see that, because $G(x,z,x',z',\omega)$ depends on the frequency, both the eigenmodes and the eigenpulsations are frequency dependent. The only difficulty is to compute these eigenmodes and eigenpulsations ; if $\epsilon_p$ and $\epsilon_f$ are small parameters, one can use pertubation expansions [6, 8] to shows that :

$$\tilde{U}_l(x,\omega)=U_l^0(x)+\epsilon_f\sum_{s=1,s\neq l}^{\infty}\frac{\omega_l^{02}}{\omega_s^{02}-\omega_l^{02}}\beta_\omega(U_l^0,U_s^{0*})U_s^0(x),\tilde{\omega}_l(\omega)=\omega_l^0(1-\imath\frac{\epsilon_p}{2\omega}+\frac{\epsilon_f}{2}\beta_\omega(U_l^0,U_l^{0*})),$$

where $U_l^0(x)=\sqrt{2/L_x}\sin(l\pi x/L_x)$ and $\omega_l^0$ are the usual l-th eigenmode and eigenpulsation of the simply supported elastic plate in vacuum. The energy exchanged with the fluid is describe by $\beta_\omega(U_l^0,U_s^{0*})=\int_0^{L_x}\int_0^{L_x}U_l^0(x')U_s^0(x')G(x,0,x',0,\omega)\mathrm{d}x'\mathrm{d}x$. Due to the various expression involved, $\beta_\omega(U_l^0,U_s^{0*})$ is given by an very simple analytical series. One shows that the solution reads

$$U(x,\omega)=\frac{1}{\rho_ph}\sum_{l=1}^{\infty}\frac{\langle F,\tilde{U}_l^*\rangle\tilde{U}_l(x,\omega)}{\omega_l^{02}-\imath\epsilon_p\omega-\omega^2(1+\epsilon_f\beta_\omega(U_l^0,U_l^{0*}))}, \tag{14}$$

where $\langle\tilde{U}_l,\tilde{U}_l^*\rangle$ is the usual inner product. The pressure radiated into the cavity is obtained by replacing the displacement given by equation 14 into equation 13.

The response of the plate and its pressure radiated into the cavity is computed using inverse time Fourier transform. Using residu integration theorem, one can shows that the response of the system involves resonance mode series expansion

$$u(x,t) = \frac{-1}{\rho_ph}\sum_{l=1}^{\infty}2\Im\left(\langle f,\hat{U}_l^{*+}\rangle\hat{U}_l^+(x)\frac{\mathrm{e}^{-\imath\hat{\omega}_lt}}{\hat{\omega}_l^+-\hat{\omega}_l^-}\right), \tag{15}$$

$$p(x,z,t) = \rho_f\int_0^{L_x}\frac{d^2u(x',t)}{dt^2}\underset{(t)}{*}g(x,z,x',0,t)\mathrm{d}x', \tag{16}$$

where $\underset{(t)}{*}$ is the usual time convolution product. The resonance modes $\hat{U}_l(x)$ and resonance pulsations $\hat{\omega}_l^{\pm}$ do not depend on the frequency and correspond to the free oscillation of the system. $\hat{\omega}_l^{\pm}$ are the zeros of $\omega_l^{02} - \imath\epsilon_p\omega - \omega^2(1+\epsilon_f\beta_\omega(U_l^0, U_l^{0*}))$ and have negative imaginary part . Each resonance mode is the corresponding eigenmode computed at the resonance frequency $\hat{U}_l^+(x) = \tilde{U}_l(x, \hat{\omega}_l^+)$. It is to be noted that to obtain the previous results, care must be taken when applying the residue integration around the first resonance frequency of the cavity which is zero for a rigid cavity. But for an excitation force with a spectrum without a continuous component, there is no particular problem.

### RESULTS

We present figure 1 two pressure fields inside the cavity 1 ms after the beginning of the excitation. The first is obtained for an excitation with $f_0$ = 500 Hz and the secondwith $f_0$ = 3000 Hz. In both cases, the maximum pressure amplitude is close to 0.014 Pa.

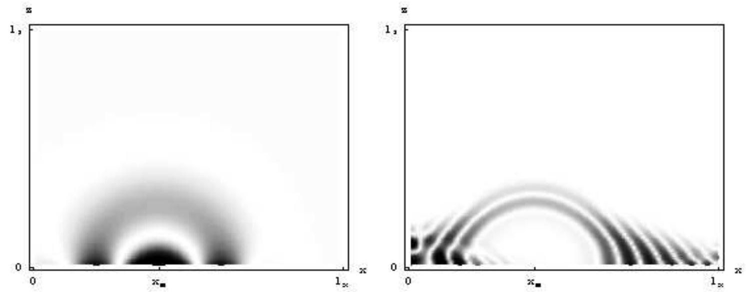

Figure 1: Pressure fields inside the cavity at 1 ms for $f_0$ = 500Hz (left) and $f_0$ = 3000Hz (right)

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